Committee machines

- Committe machines (or ensemble machines) combine the predictions of more than one classifier
- Committee machines depend on the members being reasonably accurate (better than guessing) and diverse (errors are uncorrelated)
- Majority vote and weighted majority are simple ensemble methods
- Committee machines reduce statistical error, computational error, and representational error

Generating diversity

- Given training data and a learning method, how to we generate a diverse committee of classifiers?
- We can inject randomness into the learning procedure by varying initial conditions or hyperparameters
- *Bagging* (bootstrap aggregation): randomly generate lots (25–200) of training sets by sampling the original training data with replacement majority vote
- Reduces variance, so most effective for high variance, low bias classifiers

Generating diversity

- Another strategy is to select disjoint subsets of features
- Reduces dimensionality for each classifier (important for classifiers like neural nets)
- Each resulting classifier must still be fairly accurate, so only works if features can be broken down into independent subsets
- Volcano identification (Cherkauer 1996) – 32 neural nets using 8 subsets of 119 features $\times$ 4 network sizes
- Not especially useful for NLP (but: cotraining)

Generating diversity

- We can also generate multiple classifiers by manipulating their target functions
- Error-correcting output codes are redundant encodings which allow reliable data transmission in the presence of noise
- Classifications are a kind of data transmission, and classification errors are a kind of noise
- We can use error-correcting strategies to reduce classification error (Dietterich and Bakari 1995)

Error-correcting output coding

- Suppose we have a four-class problem. We could construct a binary classifier for each class (*one-per-class* method):

c_1	1000
c_2	0100
c_3	0010
c_4	0001

- This can, of course, be reduced (*distrubuted coding*):

c_1	00
c_2	10
c_3	01
c_4	11

Error-correcting output coding

- We can also produce a redudant code, using 10 binary classifiers:

c_1	1010011100
c_2	0100011101
c_3	1001100011
c_4	1011111010

- If the bitstring produced by a test example doesn't match one of these codes exactly, choose the one with the fewest number of different bits (Hamming distance)

- For example, if our 10 binary classifiers produced the bitstring:

1010011101

we would predict class c_4

Error-correcting output coding

- If the minimum Hamming distance between any two classes is d, then the encoding can correct at least $\lfloor \frac{d-1}{2} \rfloor$ single-bit errors
- In the one-per-class case $d = 2$, and for the minimal encoding $d = 1$ (no error correction!)
- In the redundant code $d = 4$, so we can survive at least one error and still get the answer right
- To apply ECOC classification, we need to produce a suitable encoding

Error-correcting output coding

- *Row separation* increases the minimum Hamming distance between codes, and so increases the number of errors which can be corrected
- *Column separation* increases the independence of the individual classifiers, reducing the probability of simultaneous errors
- Exhaustive code for k classes uses $2^{k-1} - 1$ bits
- First class gets a code of all 1's
- The ith class gets alternating runs of 2^{k-i} zeros and ones

Error-correcting output coding

- Exhaustive code for five classes:

c_1	111111111111111
c_2	000000001111111
c_3	000011110000111
c_4	001100110011001
c_5	010101010101010

- Minimum Hamming distance is 8, and no two columns are identical or complementary

- Only really practical for $3 < k \leq 7$

Error-correcting output coding

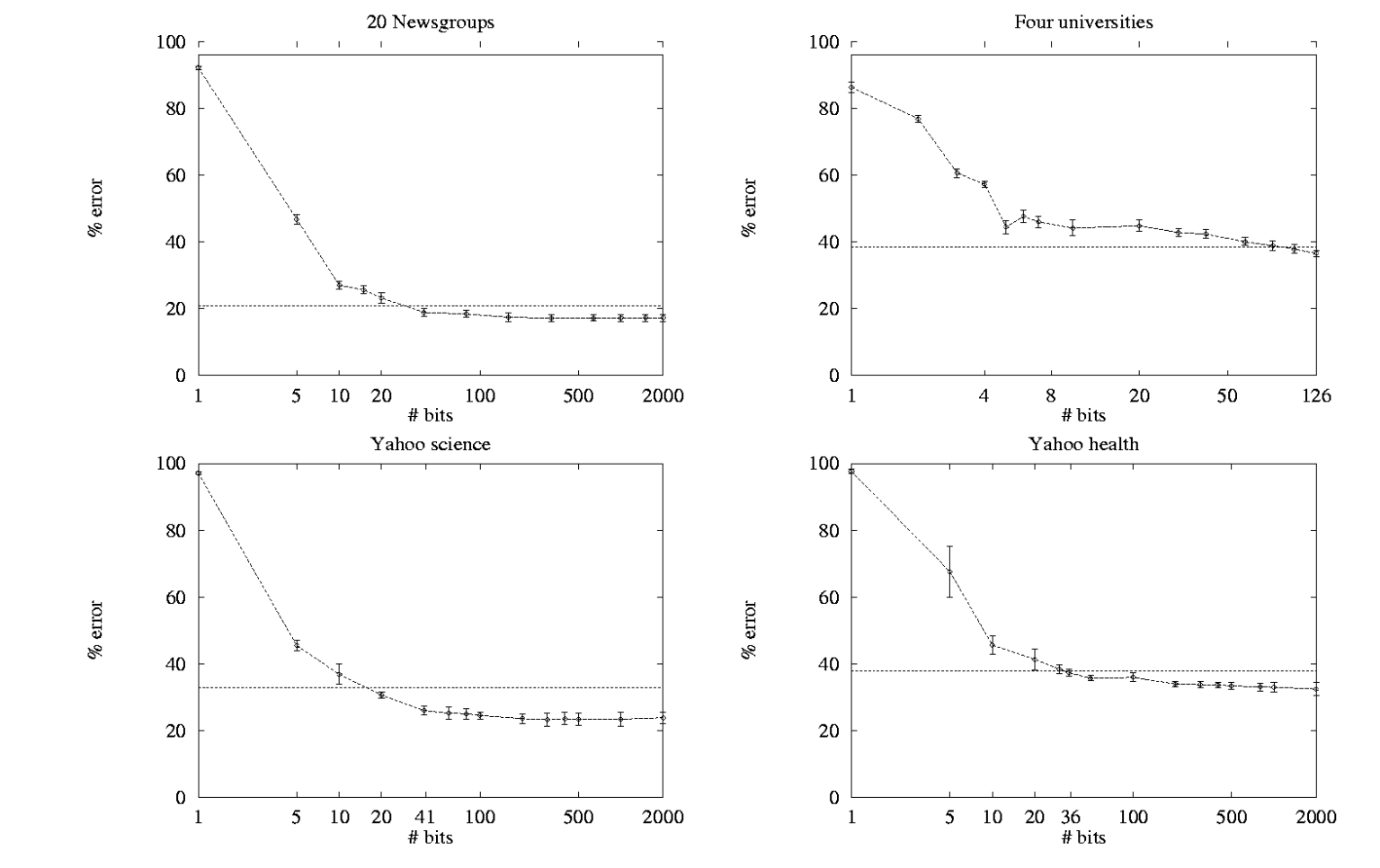
- For medium-sized problems ($8 < k \leq 11$), a subset of columns from an exhaustive code will work well
- For longer codes, randomly generated bitstrings of length L will have an average Hamming distance of $\frac{L}{2}$ (which can be heuristically improved)
- Sensible code lengths for k classes range from $\log_2 k$ to $\frac{2^k - 1}{2}$
- Assignment of classes to codes doesn't matter

Error-correcting output coding

- Each classifier learns a subset of the interclass boundaries
- Each boundary is learned by many classifiers
- Decoding is a kind of voting (and so reduces variance)
- ECOC decoding also apparently reduces bias
- Can be combined with other methods (e.g., bagging) to reduce error rate even further

Error-correcting output coding

- Applied to text classification using naive Bayes (Berger 1999):



Boosting

- The committee methods we've seen so far take reasonably accurate single classifiers and improve them
- How accurate do the base members need to be?
- Weak vs. strong ("probably approximately correct") learners
- Hypothesis Boosting Problem (Valiant 1989)– does the existence of a weak learner imply the existence of a stronger learner?
- Shapire (1990) offered an algorithm for *boosting* weak learners to strong learners

Boosting

- Shapire's proto-boosting algorithm forms the basis for a series of ensemble methods
- First, train a weak learner W_1 on N_1 examples
- Next, use that learner to filter new training examples
  - ⋆ Flip a coin
  - ⋆ If heads, use W_1 to classify new examples until one is misclassified, and add that to a new training set
  - ⋆ If tails, use W_1 to classify new examples until one is correctly classified, and add that
  - ⋆ Continue until N_1 examples have been collected.
- Now we've got a second training set (on which W_1 will have an error rate of 50%) that we use to train a second weak learner W_2

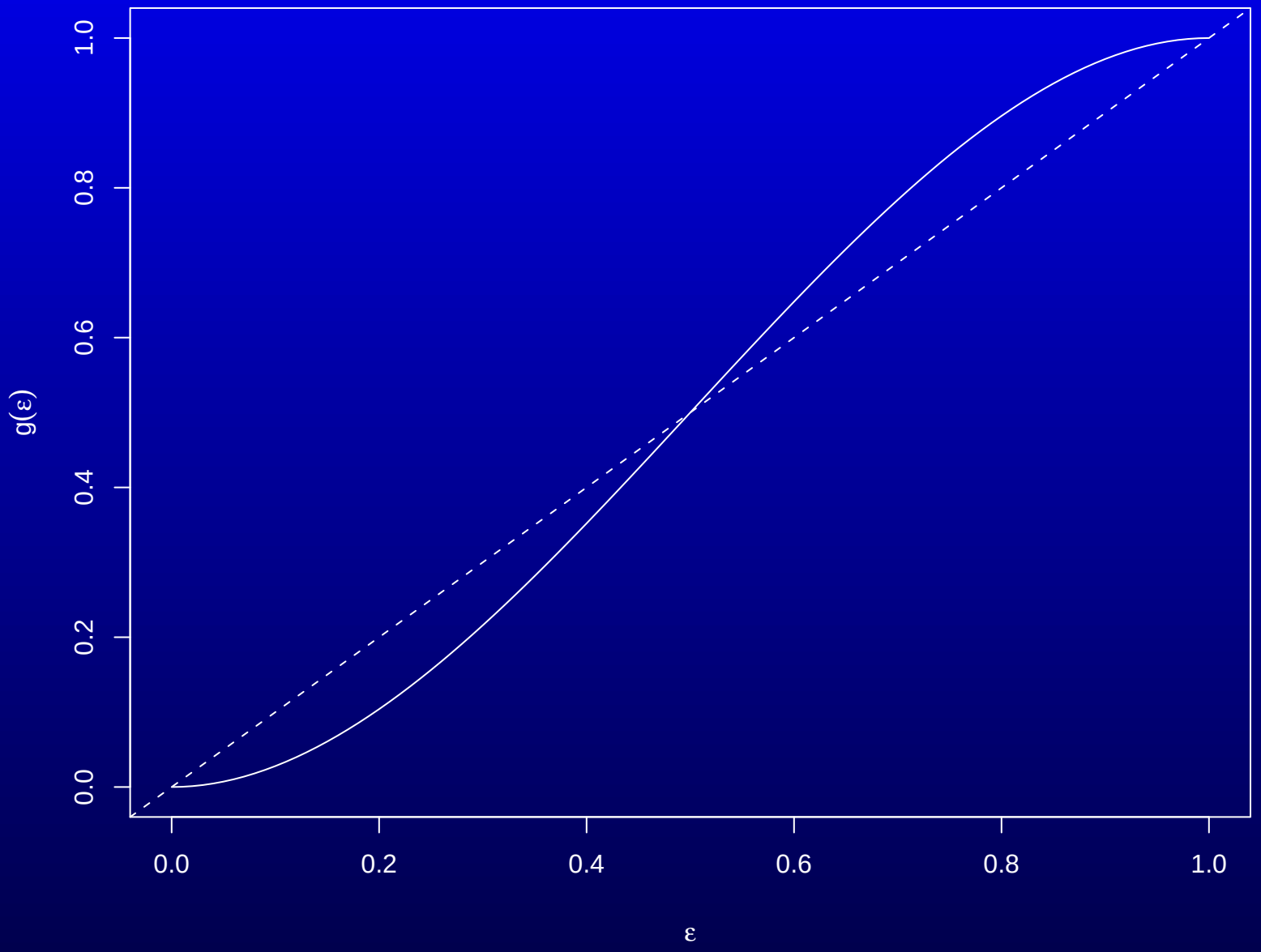
Boosting

- Finally, collect N_1 examples on which W_1 and W_2 disagree, and use them to train W_3
- To classify a new example, send it to W_1 and W_2. If they don't agree, use W_3
- If the error rate of the weak learners is ϵ, the error rate of the ensemble is bounded by:

$$g(\epsilon) = 3\epsilon^2 - 2\epsilon^3$$

- By applying the procedure recursively, the error rate can be made arbitrarily small, thus converting a weak learner into a strong learner
- In practice, this will require vast quantities of training data and so isn't very practical

Boosting



Boosting

- A family of boosting algorithms improved on Shapire's (1990) filtering algorithm
- What boosting methods have in common is that they train weak learners on different training distributions
- Unlike bagging, boosting lowers both the bias and the variance
- So, boosting may be helpful with a wide range of base learners,
- Most popular is decision stumps (high bias, low variance)

AdaBoost

- Freund and Schapire (1997) present AdaBoost (what most people mean when they refer to “boosting”)
- Suppose we have a two class problem, with features X and classes $Y \in \{-1, 1\}$, and a classifier $G : X \to \{-1, 1\}$
- The training error of the classifier is:

$$\epsilon = \frac{1}{N} \sum I(y_i \neq G(x_i))$$

AdaBoost

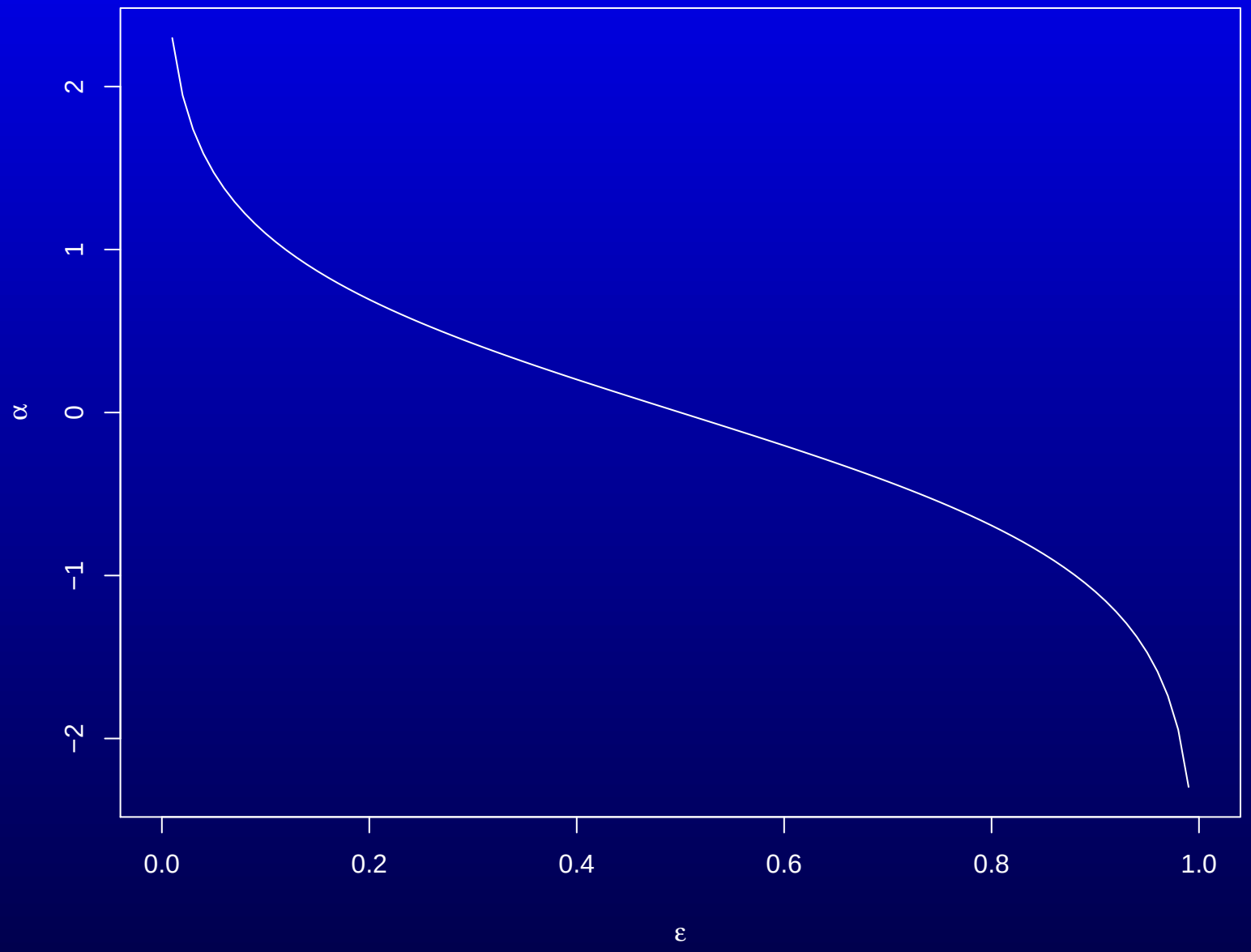
- Initialize weights w_i to $1/N,\ i = 1, \ldots, N$
- For $m = 1$ to M
  - ⋆ fit a classifier $G_m(X)$ to X using weights $w_i^{(m)}$
  - ⋆ compute training error: $\epsilon_m = \sum w_i^{(m)} I(y_i \neq G_m(x_i))$
  - ⋆ compute $\alpha_m = \frac{1}{2} \log \frac{1-\epsilon_m}{\epsilon_m}$
  - ⋆ set new weights:

$$w_i^{(m+1)} = \frac{w_i^{(m)} \exp[-\alpha_m y_i G_m(x_i))]}{Z_m},\ i = 1, \ldots, N$$

- Return

$$G(x) = \text{sign}\left[\sum_m \alpha_m G_m(x)\right]$$

AdaBoost



AdaBoost

- The training error of G is bounded by:

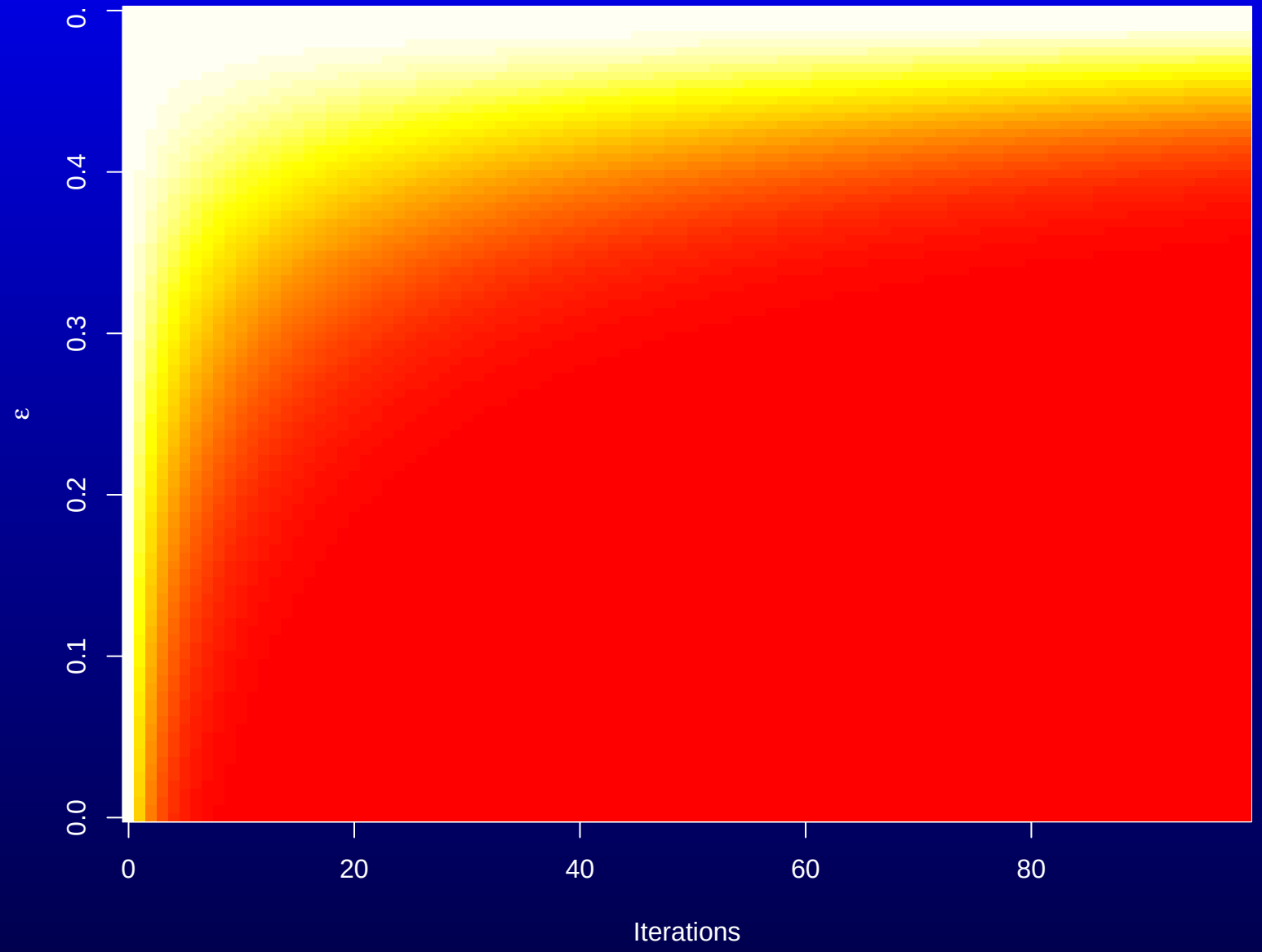
$$\begin{aligned}\frac{1}{N}\sum I(y_i \neq G(x_i)) &\leq \frac{1}{N}\sum \exp[-y_i \sum_m \alpha_m G_m(x_i)] \\ &\leq \exp[-2\sum_m \gamma_m^2]\end{aligned}$$

  where

$$\gamma_m = \frac{1}{2} - \epsilon_m$$

- As long as $\gamma_m \leq \frac{1}{2}$, the overall error rate will drop exponentially

AdaBoost



AdaBoost

- Weak learners can be very simple (decision stumps)
- AdaBoost performs extremely well, sometimes considered the best black-box learning method
- Why it works so well has been a bit of a mystery, spurring some important theoretical advances
- Extensions to multiclass classifiers and regression

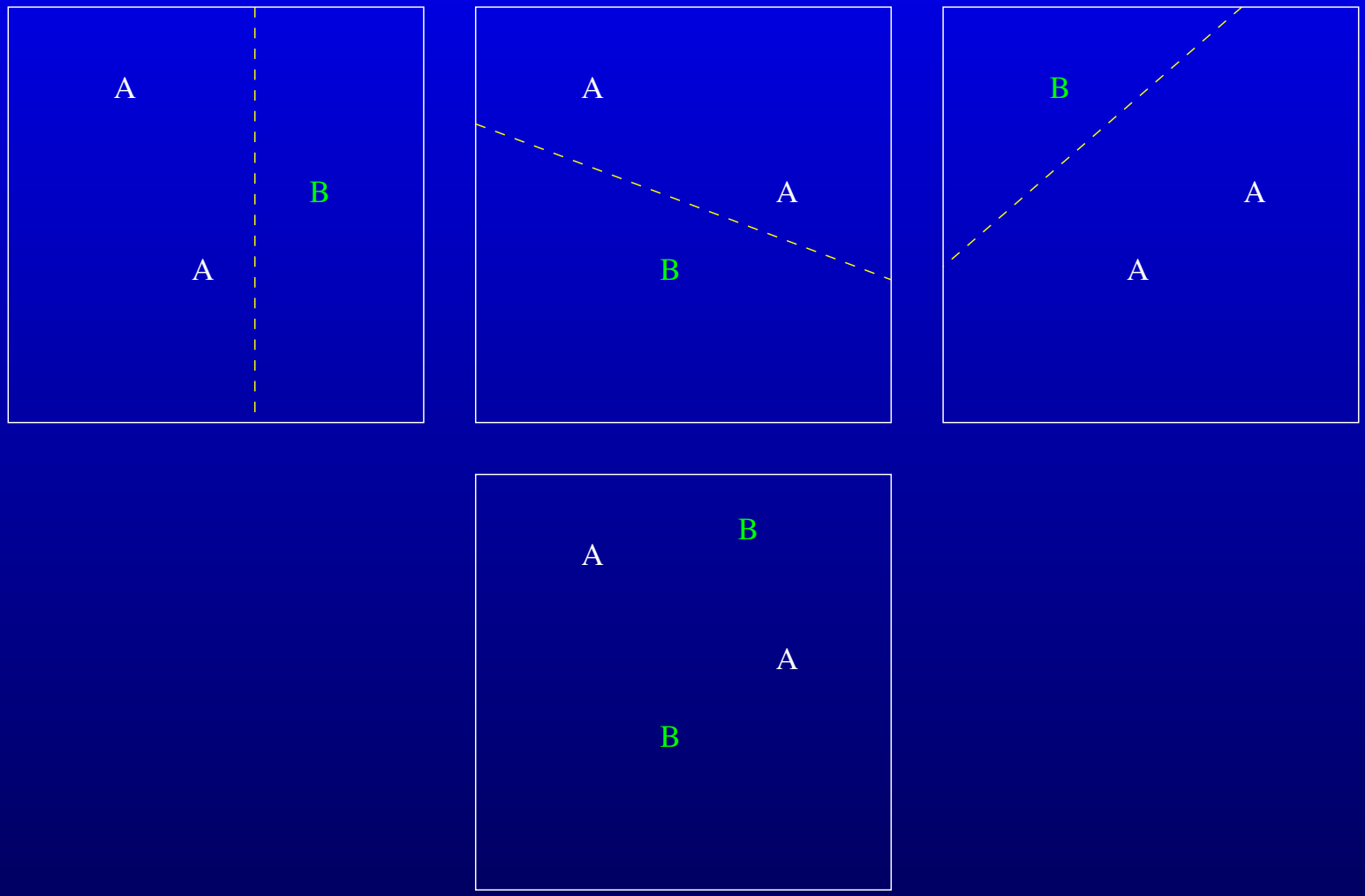
VC dimension

- Looking at training error is interesting, but doesn't tell us everything
- What we really want to know is how well the model generalizes to new data
- An intuition: simple hypotheses generalize better from training data than complex hypotheses
- How do we measure the complexity of a hypothesis space?
- The key notion is the *Vapnik-Chernovenkis (VC) dimension* a measure of how twisty the decision boundaries in a hypothesis space can be

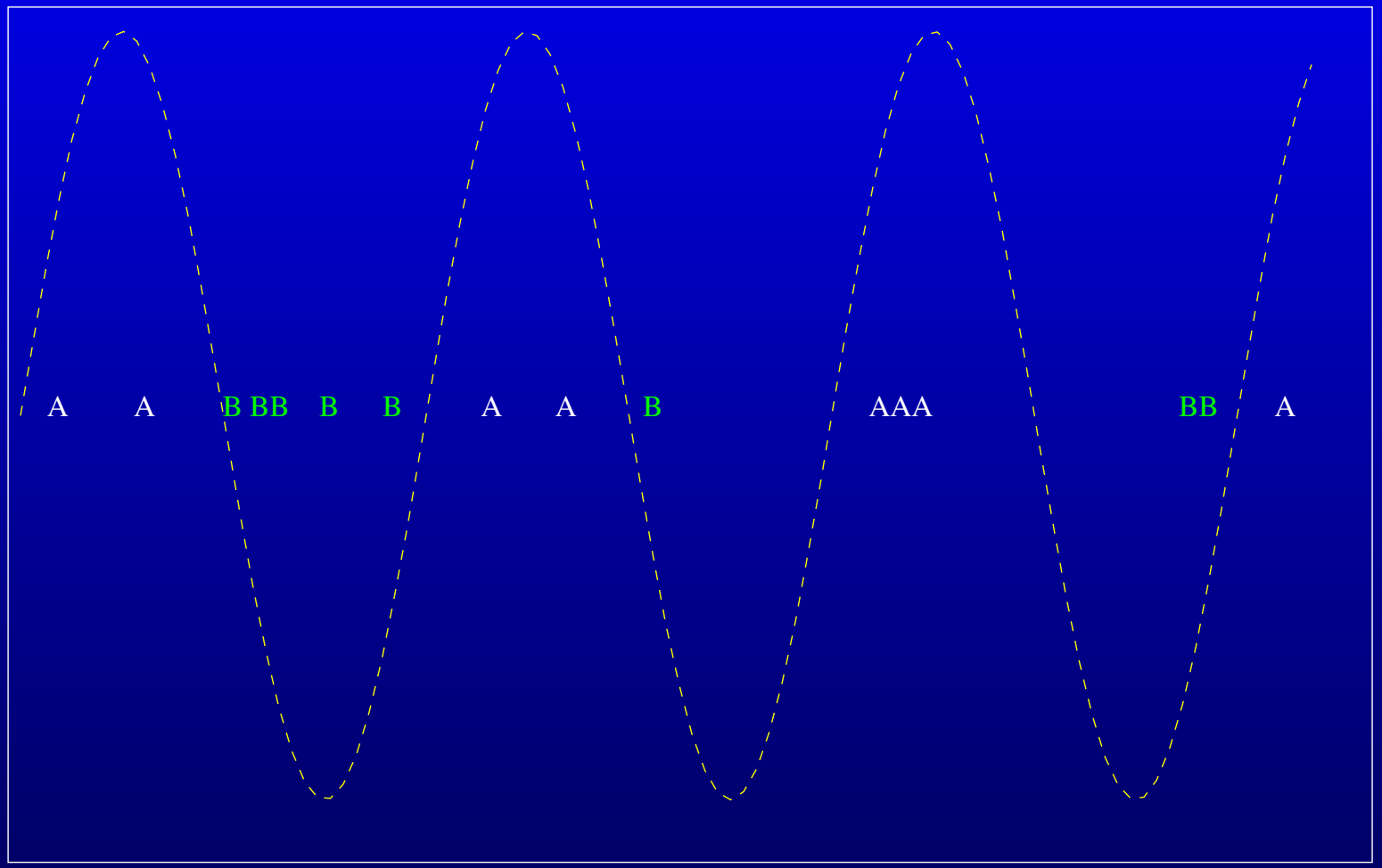
VC dimension

- The number of model parameter is a rough measure of complexity: more parameters $=$ more complex
- But compare a linear boundary $\alpha_0 + \alpha_1 x \geq 0$, with two parameters, to a boundary like $\sin \alpha x \geq 0$, with one
- The VC dimension of a class of functions $\{f(x, \alpha)\}$ is the largest number of points which can be *shattered* by members of $\{f(x, \alpha)\}$
- A set of points is *shattered* be a class of function if all possible binary class assignments can be perfectly separeted by a member of the class

VC dimension



VC dimension



VC dimension

- In general, a hyperplane in r dimensions has a VC dimension of $r + 1$
- The class $\{\sin \alpha x\}$ can shatter any number of points, and has an infinite VC dimension
- Given a classifier with a VC dimension of h and N training examples, then is $h < N$ we have with probability $1 - \delta$:

$$R[G] \leq R_{emp}[G] + \sqrt{\frac{1}{N}\left(h\left(\log\frac{2N}{h} + 1\right) + \log\frac{4}{\delta}\right)}$$

- Note that this increases with h and decreases with N

AdaBoost

- Back to AdaBoost, we have:

$$R[G] \leq R_{emp}[G] + \tilde{O}\left(\sqrt{\frac{Md}{N}}\right)$$

  where M is the number of iterations, d is the VC dimension of the base learner, and N is the number of training examples

- So, to limit generalization error, we should limit M and d

- But, a puzzle: sometimes with AdaBoost, generalization error continues to fall even when training error has reached zero!

AdaBoost

- This apparent 'superlearning' can be explained in terms of the margin, a measure of the confidence in a classification:

$$\text{margin}_G(x, y) = \frac{y \sum \alpha_m G_m(x)}{\sum |\alpha_m|}$$

- Shapire, et al. (1998) show that for any $\theta > 0$ the generalization error is at most:

$$R[G] \leq P[\text{margin}_G(x, y) \leq \theta] + \tilde{O}\left(\sqrt{\frac{d}{N\theta^2}}\right)$$

- Extra iterations can increase margins even after training error is zero
- This bound doesn't depend on M, so extra iterations need not increase the generalization error

AdaBoost

- What is AdaBoost really doing?
- It constructs a linear combination of base learners G_m which minimizes the *exponential loss*:

$$\begin{aligned} L(y, G(x)) &= \exp[-y\, G(x)] \\ &= \exp\left[-y \sum_m \alpha_m G_m(x)\right] \end{aligned}$$

- So, AdaBoost is a species of *forward stagewise additive modeling*
- More to come on this. . .

Committee machines

- Combinations of classifiers can perform better than any one classifier, so long as:
  - ⋆ each classifier is more accurate than randomly guessing
  - ⋆ the errors made by each classifier are uncorrelated
- Various strategies for producing useful committees
- Bagging is especially useful for reducing variance in high variance/low bias classifiers (like decision trees)
- Error-correcting output coding extends binary classfiers to multi-class problems, and reduces both bias and variance
- Boosting is a very powerful method, which reduces bias and variance, produces very lower error rates, and is highly resistant to overtraining